

RESERVE BANK OF INDIA

Centre for Advanced Financial Research and Learning (CAFRAL)

Seminar on Hedging Oil Requirements of Indian Oil Marketing Companies (OMCs)

**Using Structured Derivative Contracts  
to Hedge Oil-Price Exposure**

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## EHUD I. RONN — BIO

- Professor of Finance, University of Texas at Austin, 1988 – . Co-Director, Center for Energy Management and Innovation
- Director of Energy Risk Analytics at Guzman Energy, 2012 –
- 1997 – 2009: Director of the Center for Energy Finance Education and Research at the McCombs School of Business
- Ph.D. Finance, Stanford University
- Publications: Articles on investments, interest rate-sensitive instruments and energy derivatives in the academic and practitioner literature; edited **Real Options and Energy Management: Using Options Methodology to Enhance Capital Budgeting Decisions**, published in 2002 by Risk Books, London
- Additional Academic Affiliations: Member or visiting faculty at the business schools of the University of California, Berkeley, University of Chicago, European Business School, Nanyang Technological University, Dartmouth College and Fordham University
- Industry Experience:
  - Vice President, Trading Research Group, Merrill Lynch & Co., valuation and hedging of interest rate securities, 1991 – 1993
  - Practice Area Manager, Commodity Market Modeling, Morgan Stanley & Co., Jan. 2010 – Feb. 2011
- In Nov. 2004, one of fifty individuals selected by *Energy Risk* Magazine to the “Energy Risk Hall of Fame”

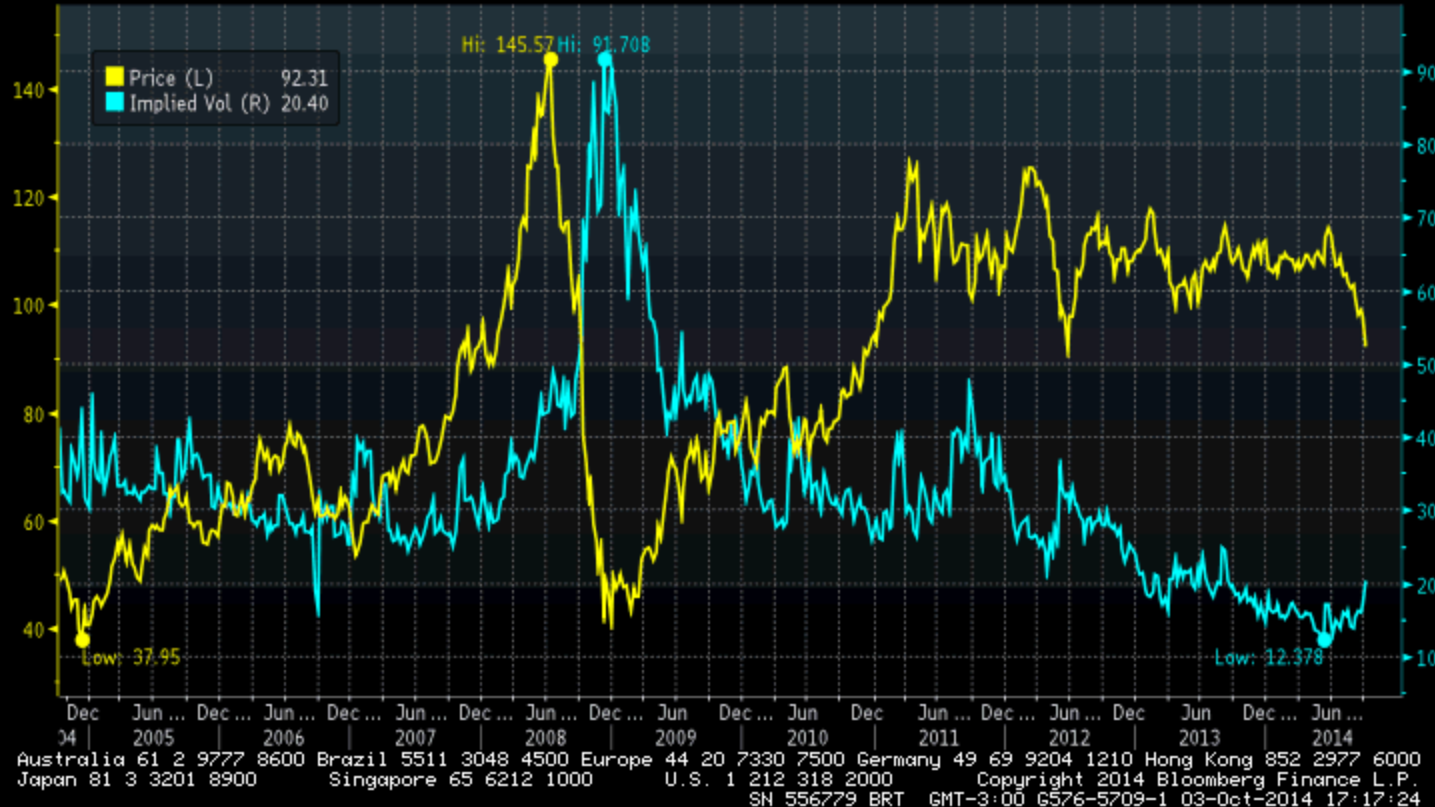
# OVERVIEW

- Making Informed Hedging Decisions: What Are Oil Markets Telling Us?
  - Oil Prices and Option-Based Implied Volatilities — The Past 10 Yrs.
  - The Level and Slope of Oil-Price Futures: Brent (North Sea) vs. Cushing (WTI)
  - Demand- and Supply-Side Oil Price Shocks:  $\text{Corr}(\text{Oil Prices, Oil Vols})$ ,  $\text{Corr}(\text{Oil Prices, Equity Prices})$
  - Is the greater Risk in Upside or Downside oil-price movements? Volatility “Smile” in the Oil Markets
- Enterprise-Wide Risk Management
  - Specifying an Objective Function for Optimal Enterprise Energy-Price Risk Management
  - Explicitly Incorporating Options in the Hedge Portfolio
- Structuring Derivative Products in Energy Markets
  - Hedging Using Linear Instruments
  - Hedging with Option Collars
  - Average-Style Options in Commodities: Case Study of a Major European Airline
  - Annual-Average Options as an Efficient Risk-Management Tool

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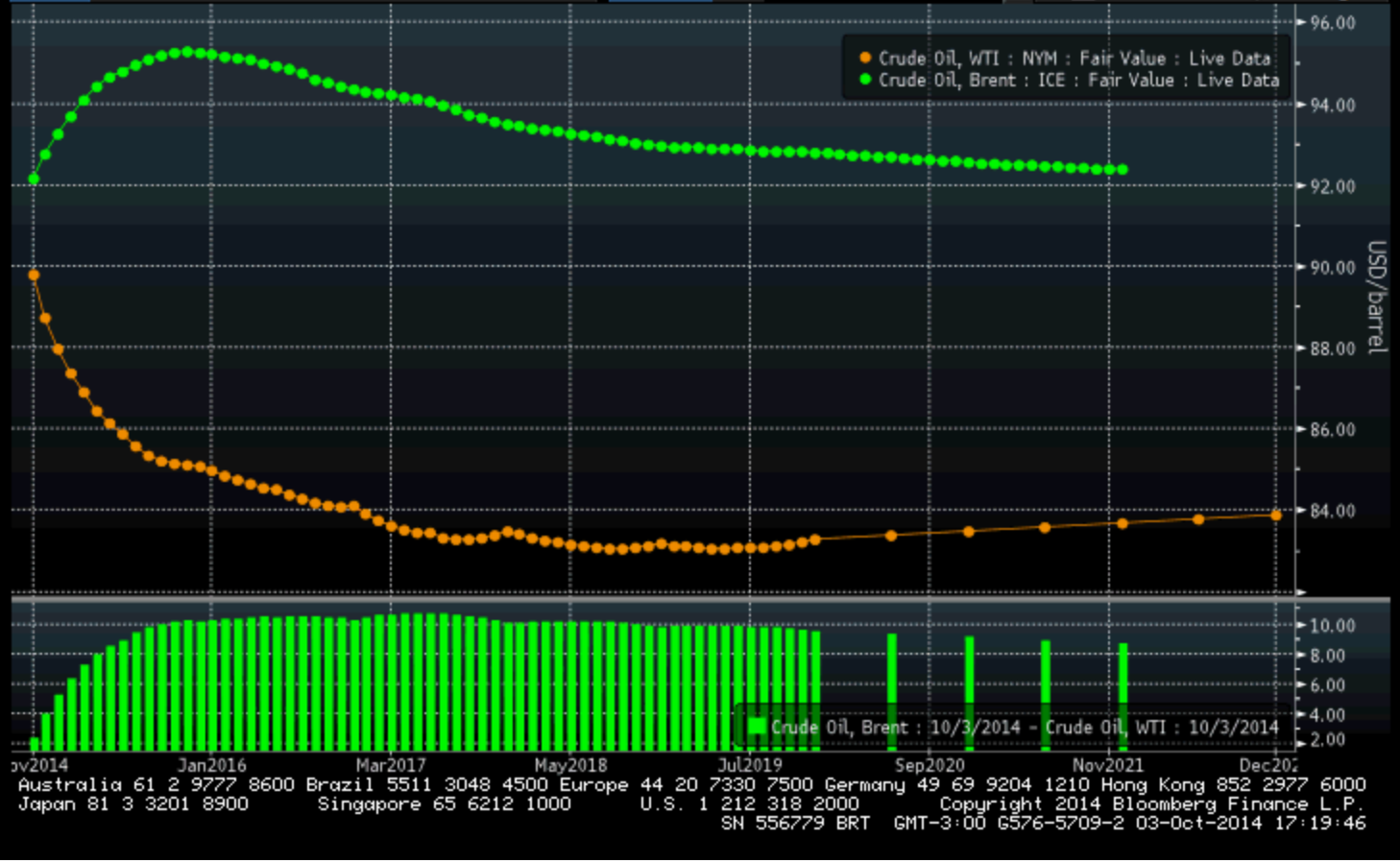
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Period Weekly Range 03-Oct-2004 - 03-Oct-2014 Ann. Factor 52 CUR LCL Statistics  
Security CO2 Comdty Hist Vol T Model CLV Price Implied Vol



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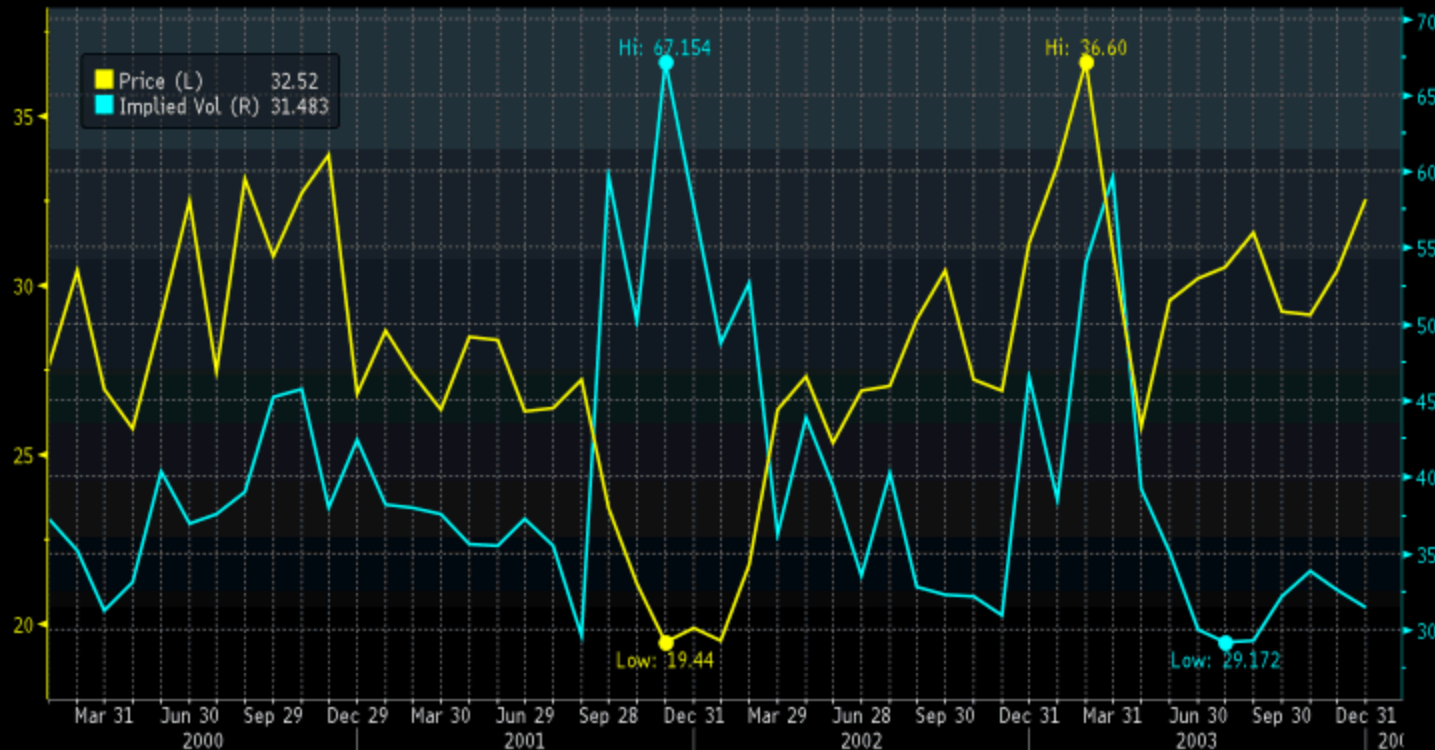


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Security CL1 COMB Comdty Hist Vol T Model CLV Price Implied Vol



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Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000  
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## Integrating Oil-Futures and Equity Markets: A CAPM-Based Expected Spot Price of Oil

- Combining the CAPM with oil futures markets,

$$\begin{aligned}\mu_i &= \beta_i (\mu_M - r) = \frac{\rho_i \sigma_i}{\sigma_M} (\mu_M - r) \\ &= \rho_i \sigma_i \frac{\mu_M - r}{\sigma_M} \equiv \rho_{it} \sigma_{it} \lambda_t,\end{aligned}\quad (1)$$

where  $\lambda_t$  is the Sharpe Ratio at date  $t$

- With respect to futures contract of maturity  $i$ ,

$$\begin{aligned}E(F_{iT}) &\equiv F_{i0} \exp\{\mu_{iT}T\} \\ &= F_{i0} \exp\{\rho_{it}\sigma_{it}\lambda_t T\} \\ \implies \frac{1}{T} \ln \left[ \frac{E(F_{iT})}{F_{i0}} \right] &= \rho_{it}\sigma_{it}\lambda_t\end{aligned}\quad (2)$$

Annualized Expected Futures Price Change  $\equiv \rho_{it} \left( \begin{array}{c} \text{Current CL}_i \\ \text{Implied Vol} \end{array} \right) \left( \begin{array}{c} \text{Current Stock Market} \\ \text{Sharpe Ratio} \end{array} \right)$

- Implication: When  $\rho_{it} < 0$  — say, because of a geopolitical crisis — the resulting  $F_{i0} > E(F_{iT})$  reflects the intuitive notion of a risk premium attributable to concerns over oil supplies reaching consumer markets

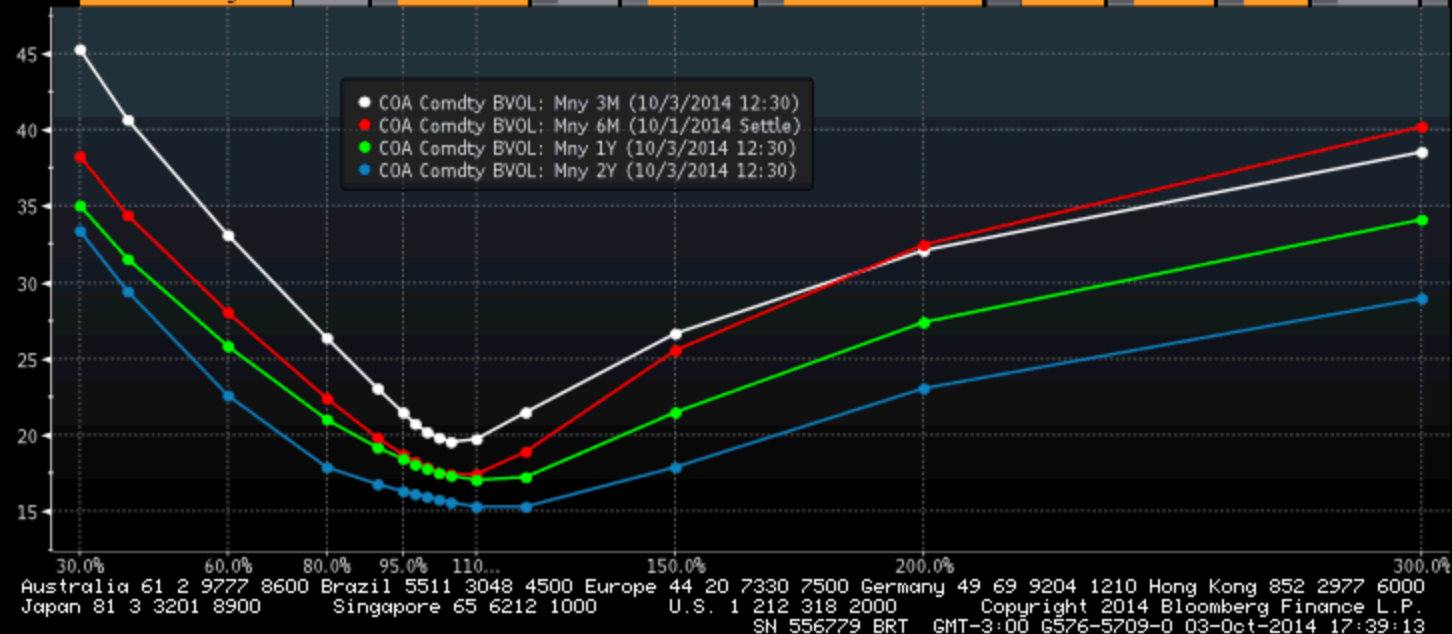
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1) Vol Table 2) 3D Surface 3) Term 4) Skew 5) Prices 7) Correlation

	Security	Data Series	Surface as of	Comparison	Mkt Vol
1.	<input checked="" type="checkbox"/> COA Comdty	BVOL 3M Mid TD	03-Oct-2014 12:30	None Abs	None
2.	<input checked="" type="checkbox"/> COA Comdty	BVOL 6M Mid 2D	01-Oct-2014 Settle	None Abs	None
3.	<input checked="" type="checkbox"/> COA Comdty	BVOL 1Y Mid TD	03-Oct-2014 12:30	None Abs	None
4.	<input checked="" type="checkbox"/> COA Comdty	BVOL 2Y Mid TD	03-Oct-2014 12:30	None Abs	None



## Calibrating Merton (1976) during the “Arab Spring of 2011”

Merton (1976) Jump-Diffusion Model: For each maturity  $T$ , including  $T = 2$  mos., the parameters of the jump process are:

$\bar{k}$  = Average amplitude of the jump process

$\delta^2$  = Variance of the jump process amplitude

$\sigma^2$  = Variance of the diffusion process

Date	Event	Country	Value of $\hat{k}_2$
Dec. 18, 2010	Self-immolation	Tunisia	−25.6%
Jan. 25, 2011	Protests in Tahrir Square	Egypt	−29.6%
Feb. 11, 2011	President Mubarak resigns	Egypt	−2.76%
Feb. 14, 2011	First contagion to Persian Gulf	Bahrain	−.26%
Feb. 19, 2011	Resignation of prime minister	Kuwait	9.44%
March 2, 2011			55.7%
March 11, 2011	Economic concessions by king	Saudi Arabia	43.2%
April 5, 2011			−5.3%

Source for Timeline: Article on the “Arab Spring,” [http://en.wikipedia.org/wiki/Arab\\_Spring](http://en.wikipedia.org/wiki/Arab_Spring)

# SUMMARY

## The Economic and Informational Role of Derivatives

- Brealey, Myers and Allen, **Principles of Corporate Finance:**

“If [financial markets are] efficient, prices impound all available information. Therefore, if we can only learn to read the entrails, security prices can tell us a lot about the future.”

Efficient capital markets — including specifically the markets for crude-oil futures and options — can be informationally-revealing

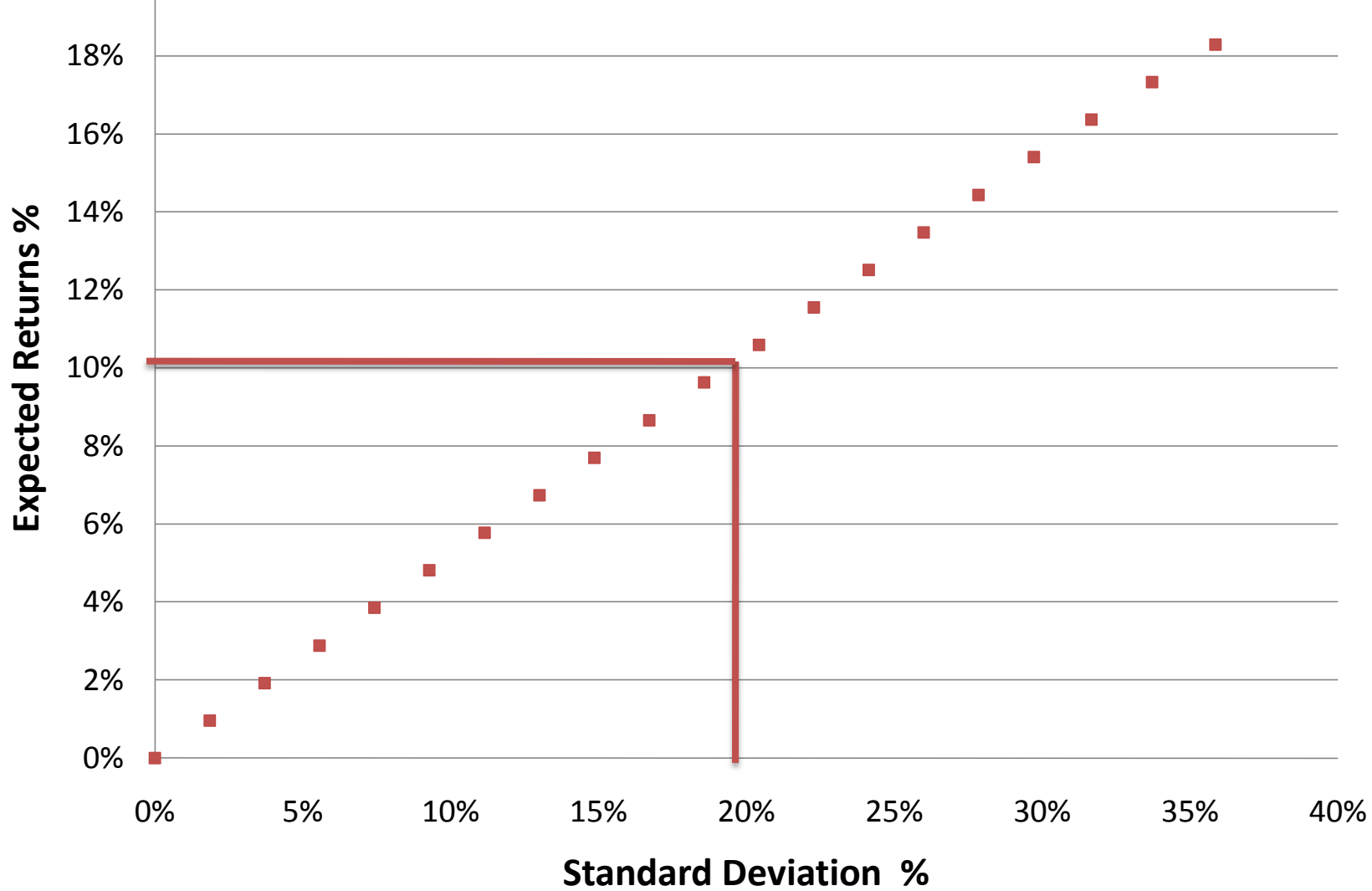
- The Challenge, as always, is to *Interpret* them. Using models developed in financial economics, such as the CAPM and Merton Jump-Diffusion, we can estimate Parameters of Interest, including
  - Expected Spot Prices and
  - Markets’ Directional Spike/Crash Concern

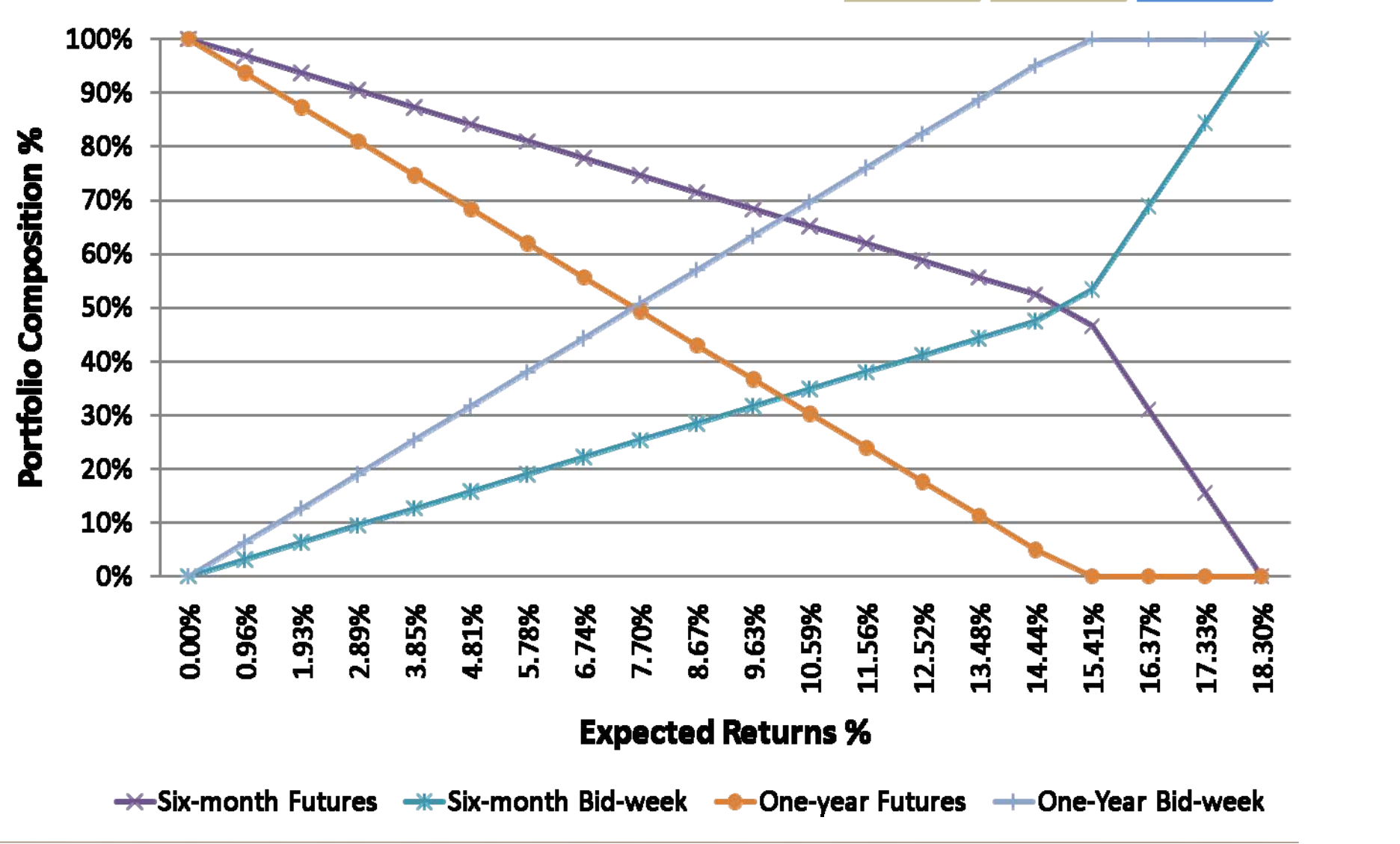
# Energy-Price Risk-Management: Motivation and Practical Implications

- Five Reasons for Energy-Price Risk Management:
  1. Understanding Risks You Are “Paid” to Take
  2. Stabilize Financial Results and Cash Flow
    - Lower Cost of Capital
    - Provide Management with “Clarity of Environment”
  3. Insure against Tail Risks
    - Protect Stakeholders against Material Hedgeable Risks
    - Can Be Source of Competitive Advantage
  4. Allow Firms to Better Plan for Their Future Capital Needs/Reduce Need to Access Capital Markets
  5. Decrease Firm’s Expected Tax Payments
- These specific objectives are not easy to model explicitly. Fortunately, in applying these rationales to the corporation, they can be shown to be equivalent to:
  1. Using mean-variance efficiency for hedging with *futures contracts*
  2. Using downside risk-aversion to motivate optimal hedging strategies involving *options*

## Application I: Mean-Variance Efficiency

- Mean-Variance Efficiency, from stock-investment Portfolio Theory: Trade-Off between Higher Mean (“Return”) and Lower Variance (“Risk”)
- Generate a risk-return profile for oil sales using:
  - Historical and implied market data
  - Modern portfolio theory
- Hedge Instruments: Futures Contracts
- Decision Tools:
  - Efficient Frontier: Choosing the Desired Trade-off between Risk and Return
  - Portfolio Composition





## Application II: Mean-Value at Risk

- Objectives:
  - Extend the discussion to Incorporating *Options* in the Hedge Portfolio
  - Corporation seeks to maximize expected earnings while simultaneously minimizing downside without affecting upside capture
- Optimization of Hedge Portfolio: Company adjusts Optimal Hedge per Desired Risk Level
  - “High” Risk: Preference for higher expected cash flows induces a firm to avoid hedging
  - “Moderate” Risk: Use of options permits downside risk protection together with maintenance of upside capture
  - “Low” Risk: Downside risk minimization becomes increasingly important motive, leading to transition from options to futures

# Implementing *Corporate-Level* Price Risk Management Policy Using Futures or Options

- For a Corporation short or long price exposure, define the objective function as:<sup>2</sup>

$$E(\tilde{E}) + \alpha E_{\text{Lower Percentile}} \quad (5)$$

- Properties of optimal solution:
  - The greater is  $\alpha$ , the more concerned is the decision-maker with the *lower* side of the earnings distribution  $\implies$  the more hedging he/she will undertake.
  - Varying  $\alpha$  from 0 (risk-neutrality) to  $\infty$  (in practice,  $\alpha$  of 1 or 2 will suffice) will provide the entire range of earnings-distribution alternatives
- Main Result. There exists a *systematic pattern to optimal hedging*: As  $\alpha$ , the degree of risk aversion, increases from zero to positive values, the company's optimal hedge proceeds from
  - *No-hedging*, to
  - *Acquiring call options* (up to 100% of expected quantity), to
  - *Replacing call options with futures contracts*
- Managerial Decision is taken upon consideration of the Expected Earnings  $E(\tilde{E})$  and the lower percentile  $E_{\text{Lower Percentile}}$

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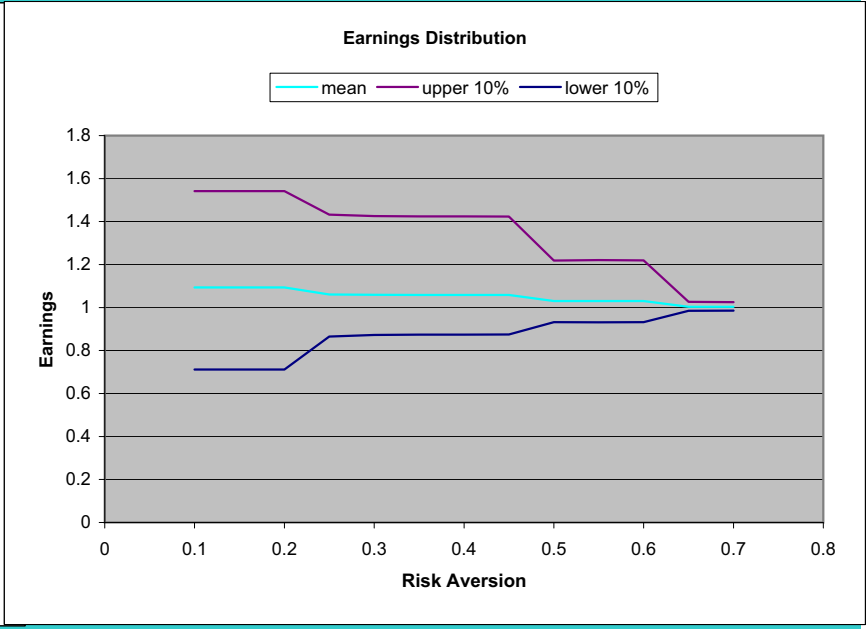
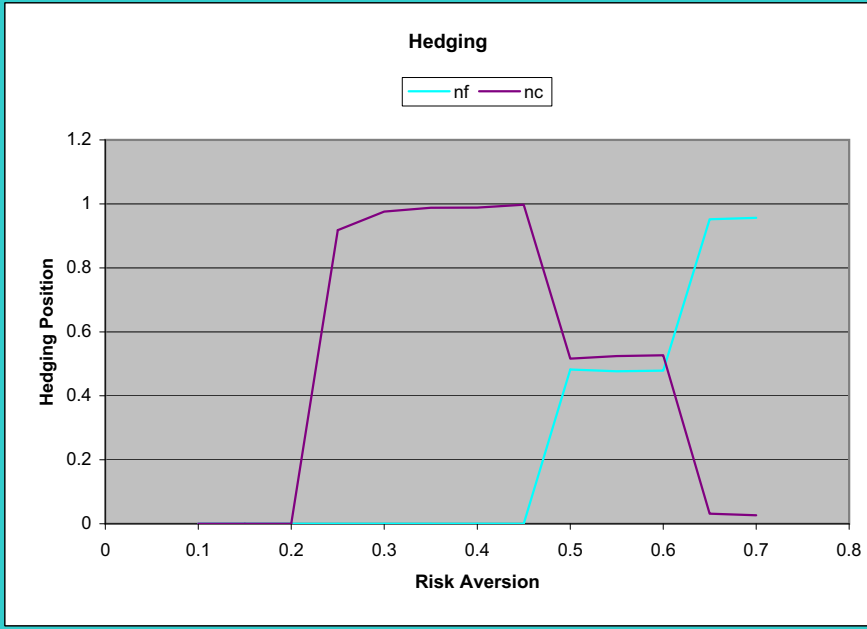
<sup>2</sup>An alternative intuitive objective function, which gives rise to the same optimal solution, is *Downside-Risk Minimization*, based on the semi-variance criterion  $E(\tilde{E}) - \alpha \sqrt{E(\max\{[E(\tilde{E}) - \tilde{E}], 0\})^2}$ .

Position in commodity	long				
Option's strike	1				
Market price of risk	0.3				
F_0	1				
Futures' drift	0.09				
Futures' volatility	0.3				
Expected quantity	1				
Quantity's volatility	0.01				
Correlation matrix	<table border="1"><tr><td>1</td><td>0</td></tr><tr><td>0</td><td>1</td></tr></table>	1	0	0	1
1	0				
0	1				

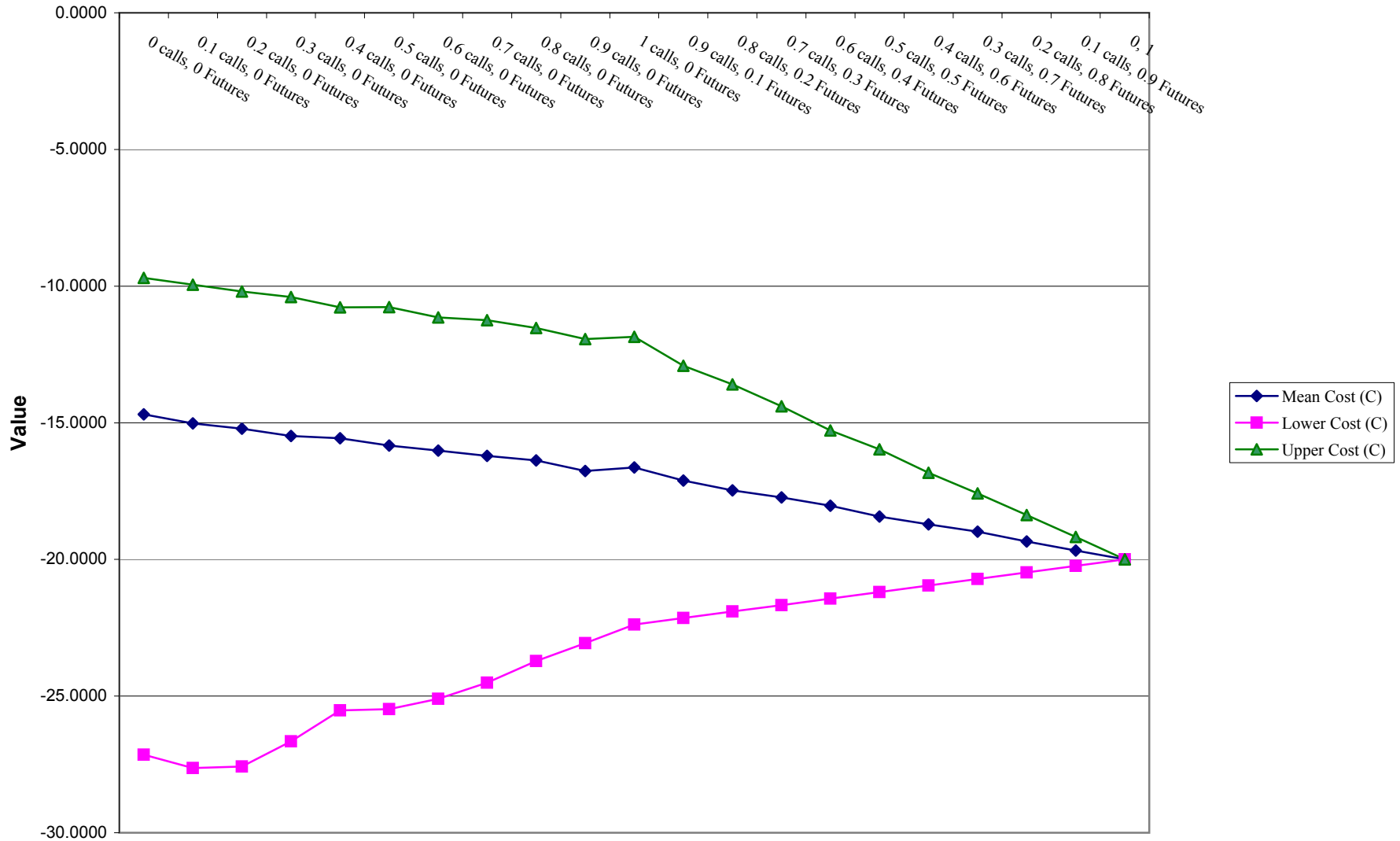
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Objective function	mean - VatRisk
Risk aversion range	0.1 0.7 0.05 0 13
Number of paths in MC	30,000
Rik free rate	0.1
Time to maturity	1
Number of time periods	1
Hedging funds	-1
Maximization algorithm	2
Horizontal axis	Risk Aversion



### Call and Future Cost



# SUMMARY

- Per Smith and Stulz (1985), Froot, Scharfstein and Stein (1993) and Grinblatt and Titman (2001), the motivation for corporate risk-management includes:
  - Reducing the costs of financial distress
  - Allowing firms to better plan for their future capital needs/reduce their need to access external capital markets
  - Improving the quality of managerial decisions by removing impact of externally-determined input/output prices
  - Improving the design of management compensation contracts/allowing firms to evaluate top executives more accurately
  - Decreasing the firm's expected tax payments
- While these criteria are difficult to model, they may be shown to correspond to corporate risk-aversion, which *can* be modeled to produce a decision framework
- Specifying an explicit risk-averse corporate objective function — e.g., mean-variance or mean-value at risk — admit specific optimal solutions and give rise to structuring an optimal derivative contracts

# Structuring Derivative Products in Energy Markets

For many market participants, the name of the game is:

Structuring — Valuation — Hedging

These represent critically important profitable market-making opportunities

- Structuring: Address client's needs (e.g., low-cost options) by providing appropriate products
- Valuation: “Reverse engineer” the structure using prices of observable futures/forwards and options

Given these prices, we can value conventional and “exotic” energy options

- Hedging: At all times, ask: Have we “spanned” the OTC product with available futures/forwards and options?

## Examples: Structured Energy Derivative Products

1. Forward/Futures contracts
2. Exchange of futures for physicals (“EFP”)
3. Swaps
4. Conventional American and European call and put options
  - (a) Monthly options
  - (b) Daily options
5. Option Collars: Combination of long call with exercise price  $K_H$ , short a put with  $K_L$ , such the customer will pay the random index  $\tilde{I}$  subject to the condition:

$$\text{Net Cost to Customer} = \begin{cases} K_H & \text{if } \tilde{I} \geq K_H \\ \tilde{I} & \text{if } K_L \leq \tilde{I} \leq K_H \\ K_L & \text{if } \tilde{I} \leq K_L \end{cases}$$

6. Average options<sup>2</sup>
7. Spread options

Product	Type of Spread
Transportation	Locational
Basis	Locational
Spark	Natural gas – electricity
Crack	Crude oil – Crude products
Frac	Natural gas – natural gas liquids

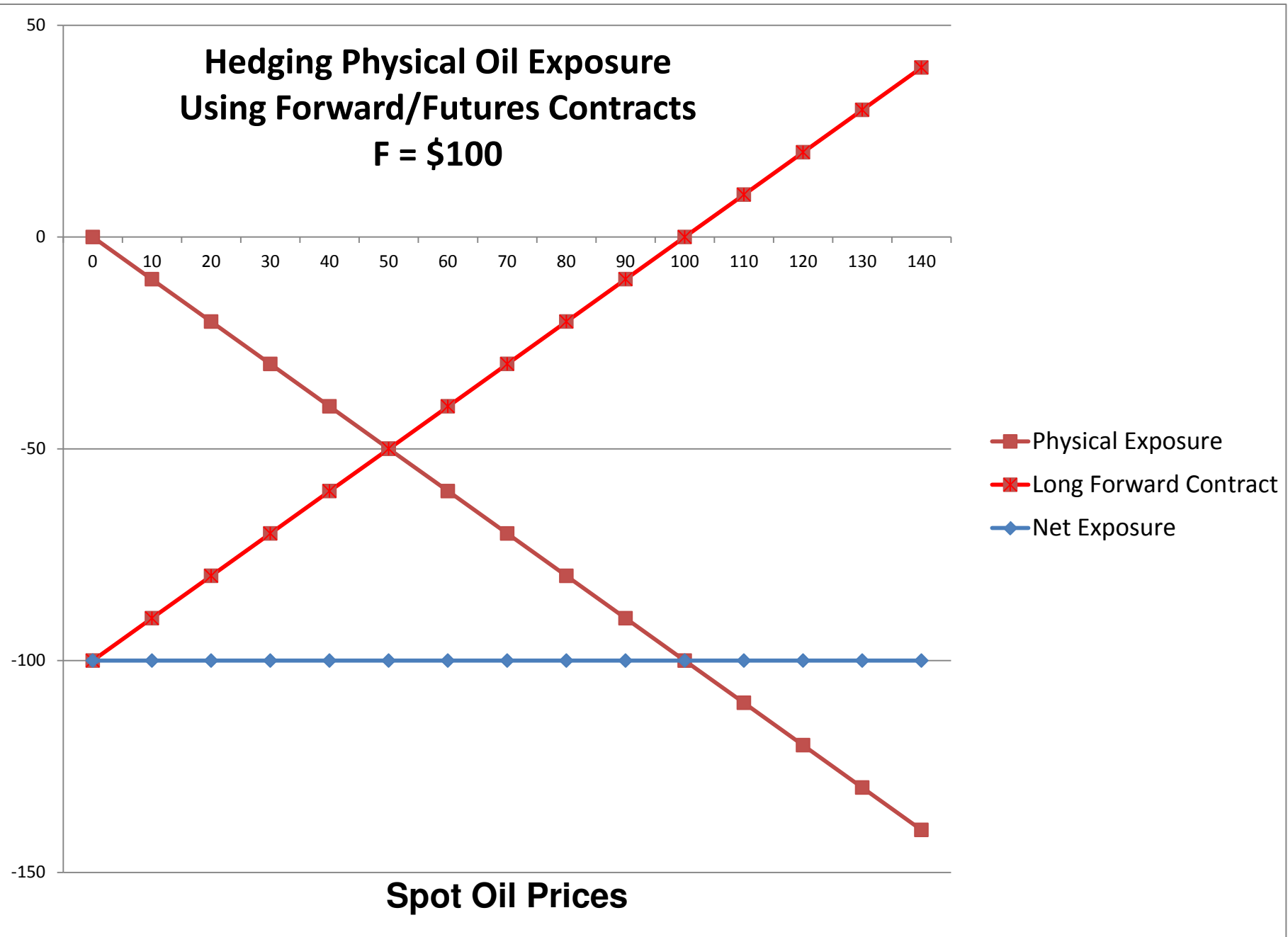
8. “Swing” options, with/without “ruthless” exercise
9. Weather derivatives

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<sup>2</sup>Motivation for Average Options:

- (a) Average prices more accurately reflect commercial-users’ true exposure
- (b) Such options are cheaper than European-style options (vol of the *average price* is lower than vol of the *spot price*)

# Hedging Physical Oil Exposure Using Forward/Futures Contracts F = \$100



## Example of Structured Energy Derivative Products: Option Collars

For a client *short* the physical product,<sup>3</sup> the Option Collar is:

- Short the physical spot
- Long call with exercise price  $K_H$
- Short a put with exercise price  $K_L$ , such the customer will pay the random index  $\tilde{I}$  subject to the condition:

$$\text{Net Cost to Customer} = \begin{cases} K_H & \text{if } \tilde{I} \geq K_H \\ \tilde{I} & \text{if } K_L \leq \tilde{I} \leq K_H \\ K_L & \text{if } \tilde{I} \leq K_L \end{cases}$$

- For a costless collar (aka “zero-cost collar”),

$$c(K_H) = p(K_L)$$

- Note implications for costless collar as  $K_H \rightarrow K_L$

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<sup>3</sup>For a client *long* the physical product, the Option Collar constitutes a combination of long physical, long a put with exercise price  $K_L$ , short a call with  $K_H$ .

## Numerical Example — Option Collar

- *Definition:* A costless collar is one in which the long call option with exercise price  $K_H$  has value equal to and offset by the short put option position with exercise price  $K_L < K_H$
- Assume:
  - $F = \$100$ ,  $T = 1$ ,  $\sigma = 40\%$ ,  $r = 6\%$ ,  $K_H = \$120$ , i.e., 20% in-the-money
  - The volatility of 40% is constant for all options expiring one year from today — i.e., for both calls and puts, and for all exercise prices.<sup>4</sup>
- *Question:*
  1. Find  $K_L$  such that the collar is costless.
  2. Describe the net cost of the commodity one year hence to the purchaser of this costless collar.
- *Solution:*

1. For the given parameters, we have

$$c(F = 100, T = 1, \sigma = 40\%, r = 6\%, K_H = 120) = 8.65$$
$$p(F = 100, T = 1, \sigma = 40\%, r = 6\%, K_L = 87) = 8.66 \cong 8.65$$

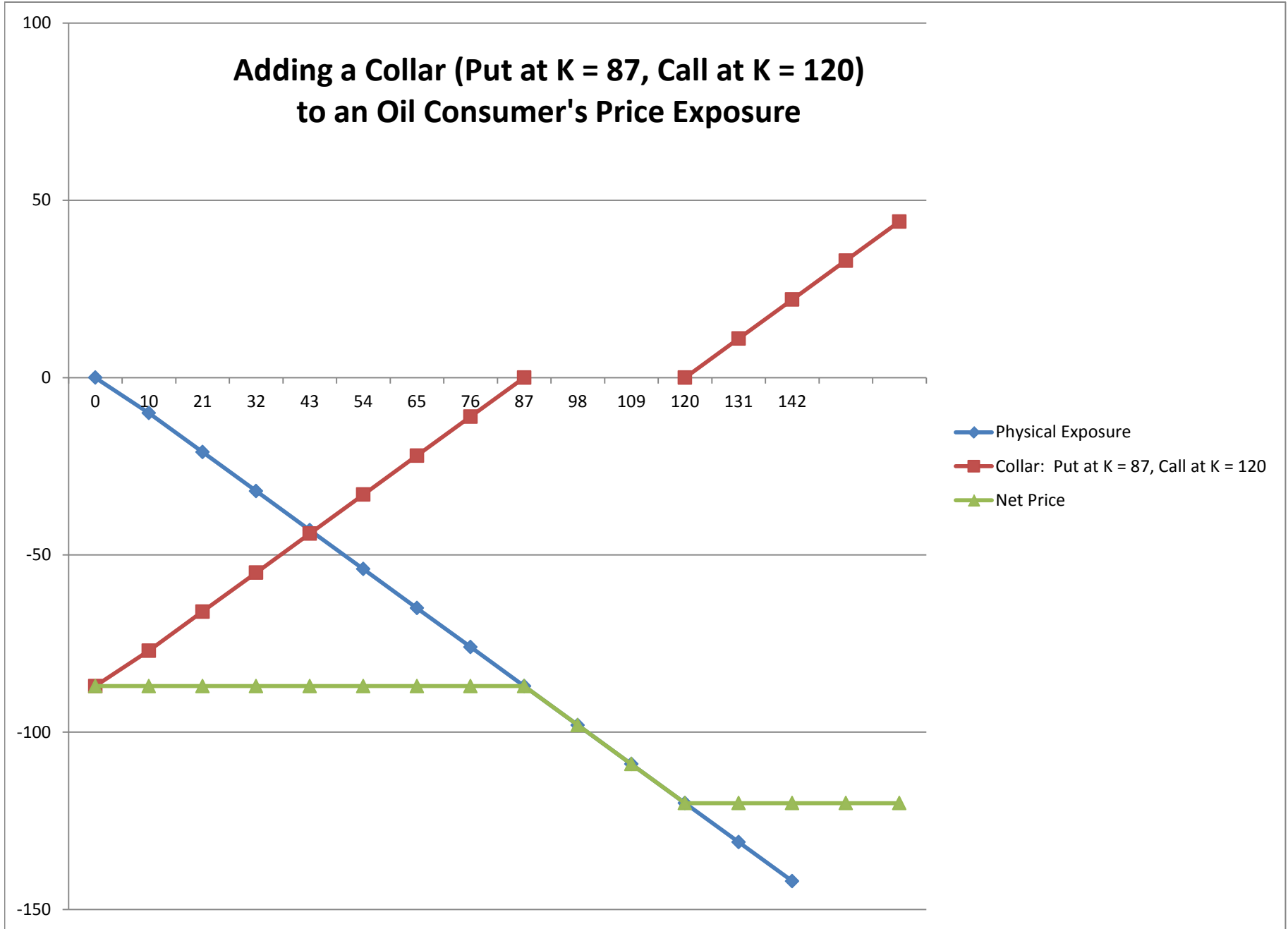
2. If the terminal value of the asset is  $I_T$ , the

$$\text{Net Cost to Customer} = \begin{cases} 120 & \text{if } I_T \geq 120 \\ I_T & \text{if } 87 \leq I_T \leq 120 \\ 87 & \text{if } I_T \leq 87 \end{cases}$$

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<sup>4</sup>This numerical example abstracts from the existence of a volatility *skew* in which vols vary by option *strike price*.

### Adding a Collar (Put at K = 87, Call at K = 120) to an Oil Consumer's Price Exposure



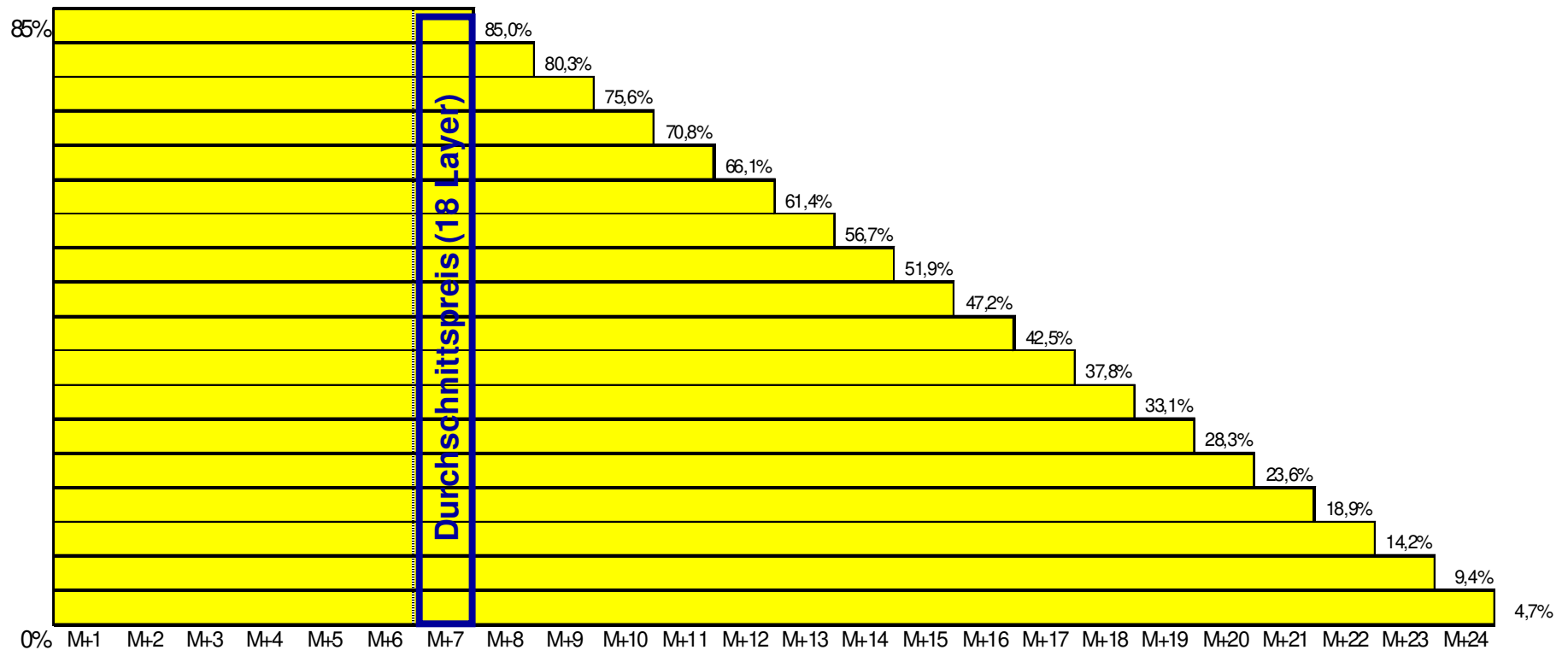
# Jet-Fuel Hedging Optimization by European Airline

1. Two principles:

- (a) Gradual hedging of 4.7% each month from month  $M-24$  to  $M-7$ . Instruments used include Swaps, Calls, Premium Collar and Four-Way
- (b) Evaluation by Plotting Sensitivity of Periodic Price (e.g., Apr. – Dec. price cond'l on realized average price)

# Layer building process based on a guideline

- Monthly hedging of 4,7% of the expected demand for M+7 up to M + 24 in crude oil
- Degree of coverage aimed for: 85 % for specific business model only

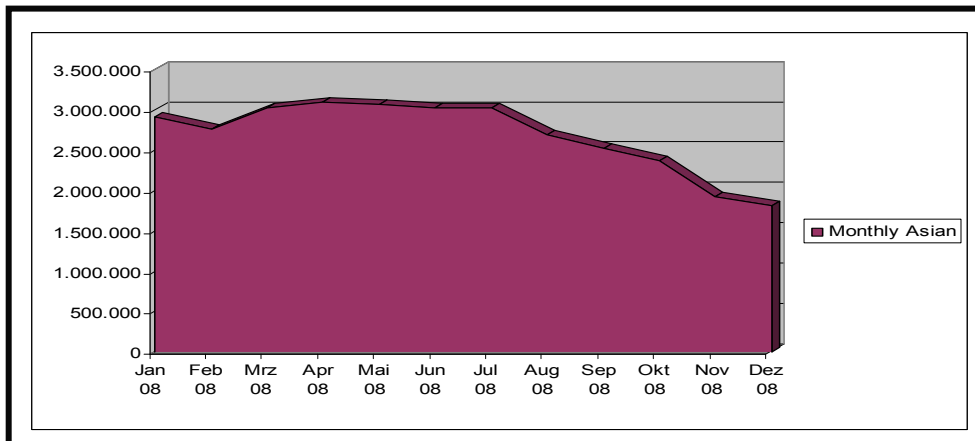


# Hedging Portfolio for 2008 – Collar Structure

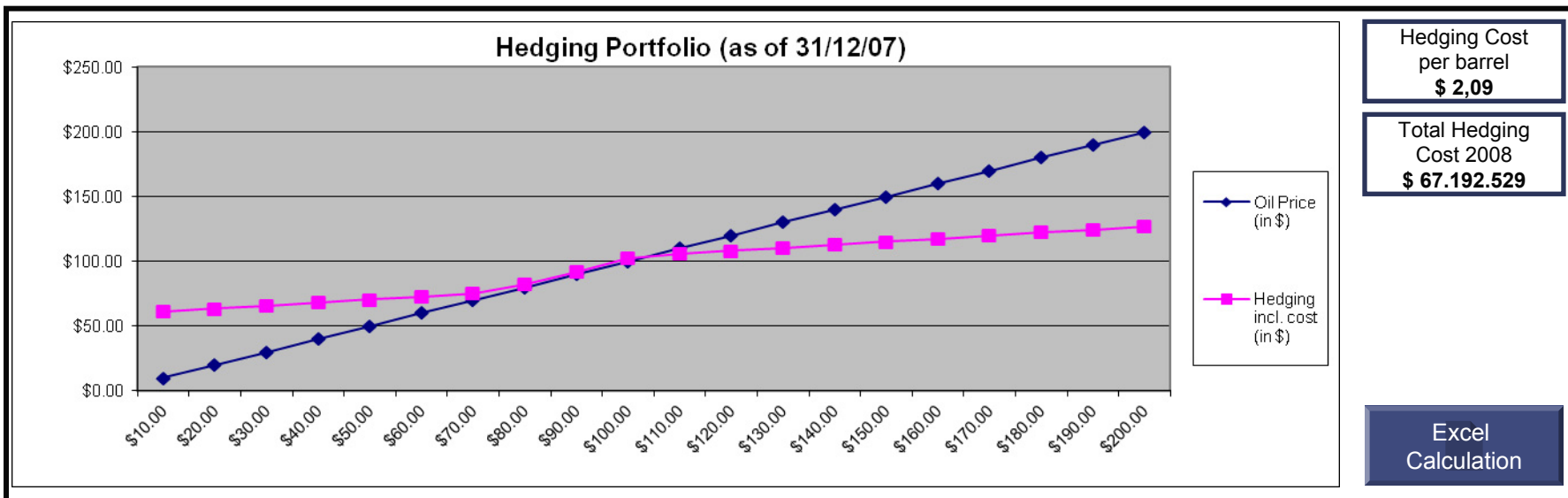
## Assumptions:

- Transaction date for all options: 12/31/2007
- Pricing of the Options using future prices and volatilities as of 31/12/2007
- Risk free rate of 3%
- Portfolio of Collars
  - > long call (110% strike price)
  - > short put (80% strike price)
- Usage of monthly Asian Options
- Constant monthly consumption of 3.500.000 barrels of oil

## Hedging Volume



## Portfolio Position “S-Curve”:



## Proposing an *Annual Average-Style Option*

- *Actual Hedge Instrument:* A strip of collars using Last-Month Averaged Options (LMAO) on the Brent futures contract, whereby the realized futures prices are averaged over their last month<sup>2</sup>
- *Proposed Hedge Instrument:* One annual-averaged price option
  - Disadvantage: Whereas a strip of Monthly-Averaged Options permits 12 distinct exercise dates, the Annual-Average is a one-time exercise (similar to the distinction between a cap and a swaption)
  - Advantage: Averaging will induce lower cost; Protection against annual-average price intact

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<sup>2</sup>In practice, Brent futures contracts mature mid-month, requiring a blending of two contracts to obtain the monthly averaged price.

## The Financial Environment, 12/31/07

Datum	Fraction of ATM	Value
Futures Price	100%	\$92.37
Call Strike Price	110%	\$101.76
Put Strike Price	80%	\$74.01

### Range of Vanilla Vols

(Jan. 2008 to Dec. 2008) 32.7% to 21.6%

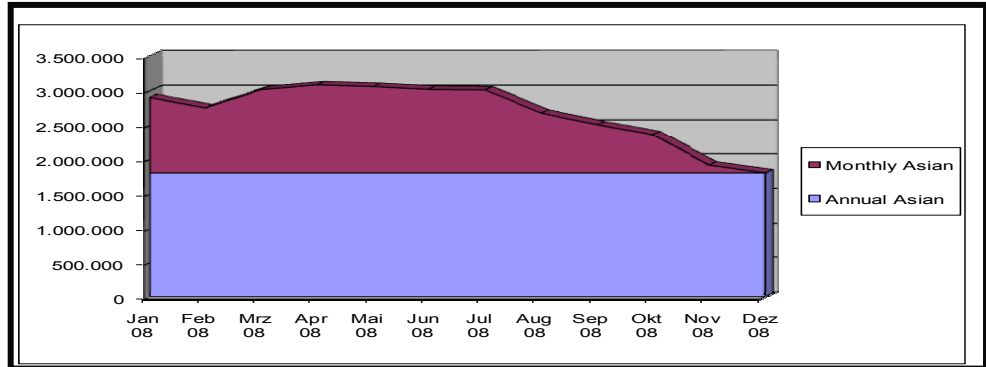
Annual Arithmetic Volatility 14.07%

# Hedging Portfolio – Min Annual Options (110/80)

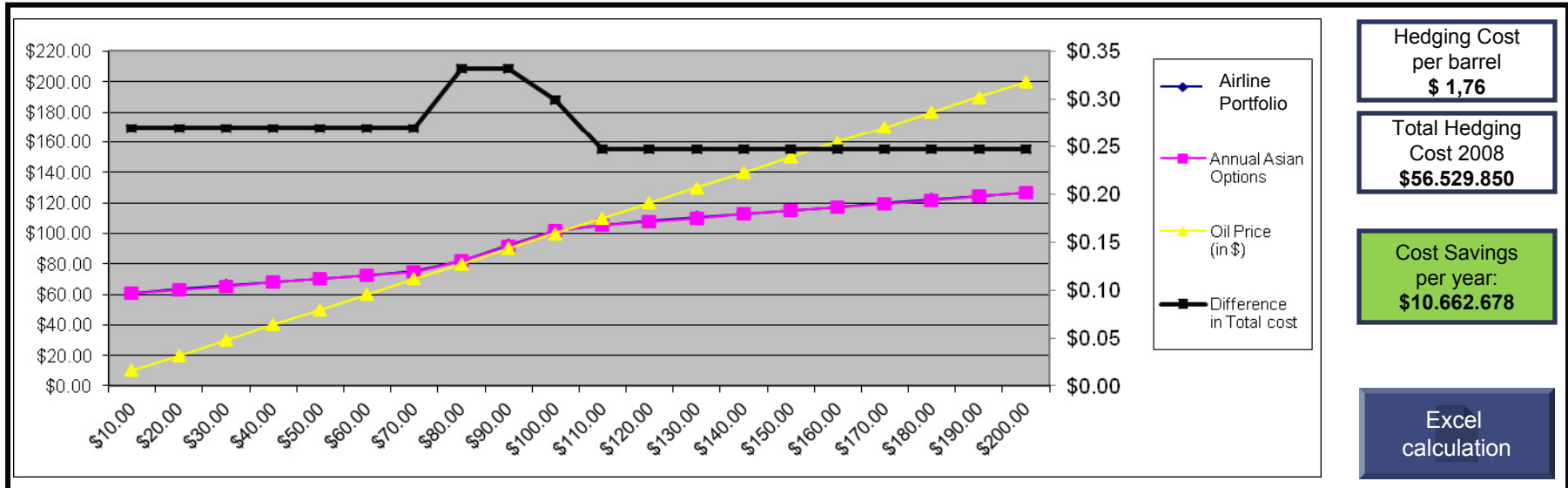
## Assumptions:

- Usage of **annual** Asian Options (1.807.000 barrel oil each month equally to hedged position of Dec 08)
  - > long call (110% Strike, \$101,67)
  - > short put (80% Strike, \$ 73,94)
- Usage of **monthly** Asian Options (difference between hedged position for Dec 08 and individual monthly hedge volume)
  - > long call (110% Strike)
  - > short put (80% Strike)

## Hedging Volume



## Portfolio Position "S-Curve":



## Commodity Futures Information (Brent Crude Oil Market Data as of Mar 11, 2013)

	May'13	Jun'13	Jul'13	Aug'13	Sep'13	Oct'13	Nov'13	Dec'13	Jan'14	Feb'14	Mar'14	Apr'14
<b>Futures Price</b>	109.58	108.97	108.37	107.78	107.11	106.49	105.96	105.44	104.96	104.49	104.03	103.58
<b>Implied Volatility</b>	18.05%	19.68%	19.95%	20.41%	20.47%	20.81%	21.06%	20.98%	21.04%	20.81%	20.78%	20.64%

## Option Pricing

Option Pricing Parameters

	May'13	Jun'13	Jul'13	Aug'13	Sep'13	Oct'13	Nov'13	Dec'13	Jan'14	Feb'14	Mar'14	Apr'14
<b>Last Month Avg Calls</b>	3.461	4.286	4.832	5.313	5.594	5.927	6.251	6.446	6.676	6.782	6.941	7.040
<b>Std European Calls</b>	4.183	4.885	5.326	5.748	5.981	6.283	6.582	6.752	6.964	7.050	7.194	7.280

	Price (\$)	Implied Volatility
Correlation Matrix		
Implied Volatilities		
<b>Annual-Average Style Option (Calls)</b>	4.99	11.79%

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## Option Comparison

	Annual-Average Option	Strip of Last-Month Average Option	Strip of Standard European Options
<b>Price of 1000 Barrels (\$)</b>	4.99	5.8	6.19
<b>Relative to Annual-Average Style</b>	100%	116.07%	123.88%
<b>Savings of 1000 Barrels (in \$M)</b>		0.8	1.19

## SUMMARY

- For Commercial users, Energy Derivatives are not “exotic for exotic’s sake”:
  - They have a natural *raison d’être*
  - They address specific risk exposures in an efficient manner
- As hedge instruments, Annual-Average Options have the triple merit:
  1. They more accurately reflect commercial-users’ true exposure
  2. Given corporate planning horizons, they appear to Span an Optimal Length of Time
  3. Their Averaging Property Reduces their Cost

## APPENDICES

**A Analytics of Managing Long or Short Energy Price- and Quantity-Exposure at the Corporate Level**

**B Consumer Three-Way Payoff**

# Optimizing a Hedging Strategy by Energy End-Users

Two Optimization Methodologies for Energy Consumer and Producer End-Users:

1. Mean-Variance: Optimal Use of Futures Contracts
2. Mean-Value-at-Risk: Optimal Use of Futures *and* Options

## Intuition and Analytics of Mean-Variance Optimization: Modeling the Commodity Producer’s Hedging Optimization

### 1. Intuition:

- (a) Model the expected rate of return for a producer hedging with short position in futures contracts
- (b) Create a *tension* between expected rate of return and std. dev.: The less risk the producer is willing to take, the lower the expected rate of return on their portfolio of physicals-futures. This creates an intuitively-plausible tradeoff, in that the hedge portfolio is neither 0 nor 100% of expected output

### 2. Analytics:

- (a) Model the notion of a “Commodity Market Price of Risk,” a risk premium measure which corresponds to the Sharpe Ratio in equities
- (b) Consider a producer facing a two-period  $t = 0.5$  and  $t = 1$  optimization, with expected output  $Q_t$ :

$$V = \sum_{t=1/12}^1 \frac{Q_t F_{0t}}{(1+r)^t} \quad (1)$$

where

$V$  = Value of Cash Flows to be Received at six-mos. and 1-yr.

$Q_t$  = Fixed Quantity at time  $t$ ,  $t = 0.5, 1$

$F_{0t}$  = Current (time 0) futures price for contract maturing at time  $t$ ,  $t = 1/2, 1$

$r$  = Discount rate

- (c) Let  $\tilde{V}$  stand for uncertain value under incomplete hedging. Since the producer is long energy, the hedge uses  $n_t$  *short* futures contracts ( $0 \leq n_t \leq Q_t$ ):<sup>1</sup>

$$\tilde{V} = \sum_{t=1/12}^1 \frac{Q_t \tilde{F}_t + n_t (F_{0t} - \tilde{F}_t)}{(1+r)^t} \quad (2)$$

- (d) We can now compute the expected value and its variance:

$$E(\Delta \tilde{V}) = \lambda \left[ \frac{(Q_{0.5} - n_{0.5}) F_{0,0.5}}{(1+r)^{0.5}} 0.5 \sigma_{0.5} + \frac{(Q_1 - n_1) F_{0,1}}{1+r} \sigma_1 \right] \quad (3)$$

where

$\lambda$  = Market Price of Risk for this commodity

$\sigma_t$  = Annualized implied volatility for date  $t$

Based on past experimentation (to obtain plausible portfolio values<sup>2</sup>), we use values of  $\lambda$  in the range [0.3, 0.8].

$\text{Var}(\Delta \tilde{V})$  is given by:

$$\begin{aligned} \text{Var}(\Delta V) = & \left[ \frac{(Q_{0.5} - n_{0.5}) F_{0,0.5}}{(1+r)^{0.5}} \right]^2 0.5 \sigma_{0.5}^2 + \left[ \frac{(Q_1 - n_1) F_{0,1}}{(1+r)} \right]^2 \sigma_1^2 \\ & + 2 \left[ \frac{(Q_{0.5} - n_{0.5}) F_{0,0.5}}{(1+r)^{0.5}} \frac{(Q_1 - n_1) F_{0,1}}{(1+r)} \right] \rho_{0.5,1} \sqrt{0.5} \sigma_{0.5} \sigma_1 \end{aligned} \quad (4)$$

- (e) To proceed to the optimization, we state things in terms of *rates of return*:

$$E\left(\frac{\Delta \tilde{V}}{V}\right) = \frac{1}{V} E(V) \quad (5)$$

$$\text{Std. Dev.}\left(\frac{\Delta \tilde{V}}{V}\right) = \frac{1}{V} \text{Std. Dev.}(\Delta \tilde{V}) \quad (6)$$

Formally, the optimization surface is obtained by solving the optimization problem

$$\min_{\{n_{0.5}, n_1\}} \text{Std. Dev.}\left(\frac{\Delta \tilde{V}}{V}\right) \quad (7)$$

$$\text{subject to } E\left(\frac{\Delta \tilde{V}}{V}\right) = c,$$

which “sketches out” the optimization frontier as  $c$  varies

<sup>1</sup>Any expression with a tilde (“ $\sim$ ”) is random as of time-0.

<sup>2</sup>We seek portfolios that provide latitude for both hedging and non-hedging.

## Definition of Earnings for Corporations Short or Long Price Exposure

- Definition of Earnings (for a Corporation *Short* Prices):

$$\tilde{E} = -\tilde{P}\tilde{Q} + n_F(\tilde{P} - F) + n_C \max\{0, \tilde{P} - K\} - n_C(1+r)^T C(K) \quad (3)$$

where

Notation	Definition
$E(\tilde{P})$	Current expectation of average price for month $T$
$E(\tilde{Q})$	Current expectation of total quantity consumed in month $T$
$\sigma_Q, \sigma_P$	Proportional volatility for $\tilde{Q}$ and $\tilde{P}$
$\rho_{P,Q}$	Correlation coefficient between $\ln P$ and $\ln Q$
$F$	Forward price for month $T$
$C(K)$	Call option with given strike price $K$
$T$	Time to expiration
$r$	Riskfree rate of interest

- For a corporation *long* prices, the analogous earnings statement is

$$\tilde{E} = \tilde{P}\tilde{Q} - n_F(\tilde{P} - F) + n_{\text{Put}} \max\{0, K - \tilde{P}\} - n_{\text{Put}}(1+r)^T \text{Put}(K) \quad (4)$$

# Implementing *Corporate-Level* Price Risk Management Policy Using Futures or Options

- For a Corporation short or long price exposure, define the objective function as:<sup>2</sup>

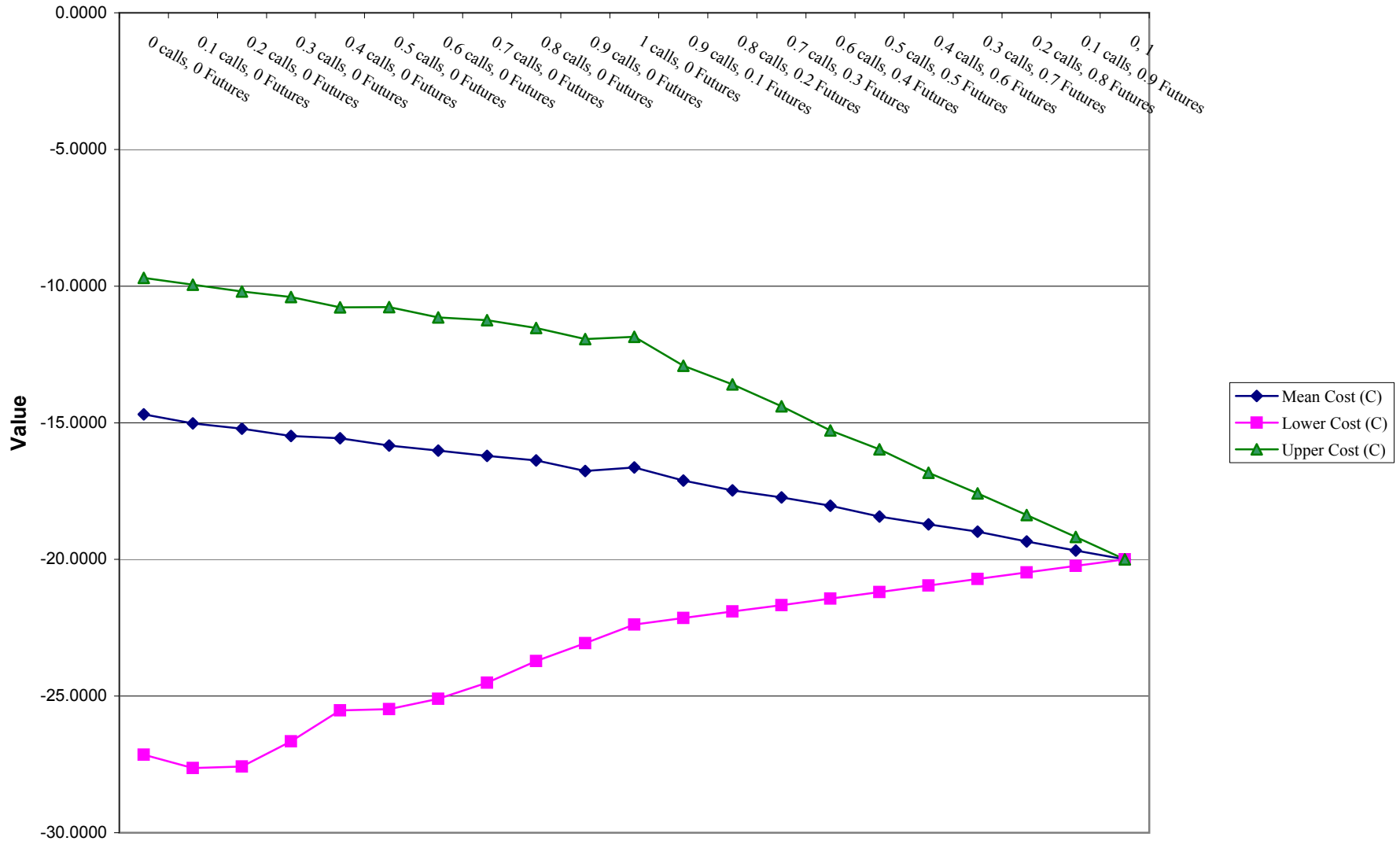
$$E(\tilde{E}) + \alpha E_{\text{Lower Percentile}} \quad (5)$$

- Properties of optimal solution:
  - The greater is  $\alpha$ , the more concerned is the decision-maker with the *lower* side of the earnings distribution  $\implies$  the more hedging he/she will undertake.
  - Varying  $\alpha$  from 0 (risk-neutrality) to  $\infty$  (in practice,  $\alpha$  of 1 or 2 will suffice) will provide the entire range of earnings-distribution alternatives
- Main Result. There exists a *systematic pattern to optimal hedging*: As  $\alpha$ , the degree of risk aversion, increases from zero to positive values, the company's optimal hedge proceeds from
  - *No-hedging*, to
  - *Acquiring call options* (up to 100% of expected quantity), to
  - *Replacing call options with futures contracts*
- Managerial Decision is taken upon consideration of the Expected Earnings  $E(\tilde{E})$  and the lower percentile  $E_{\text{Lower Percentile}}$

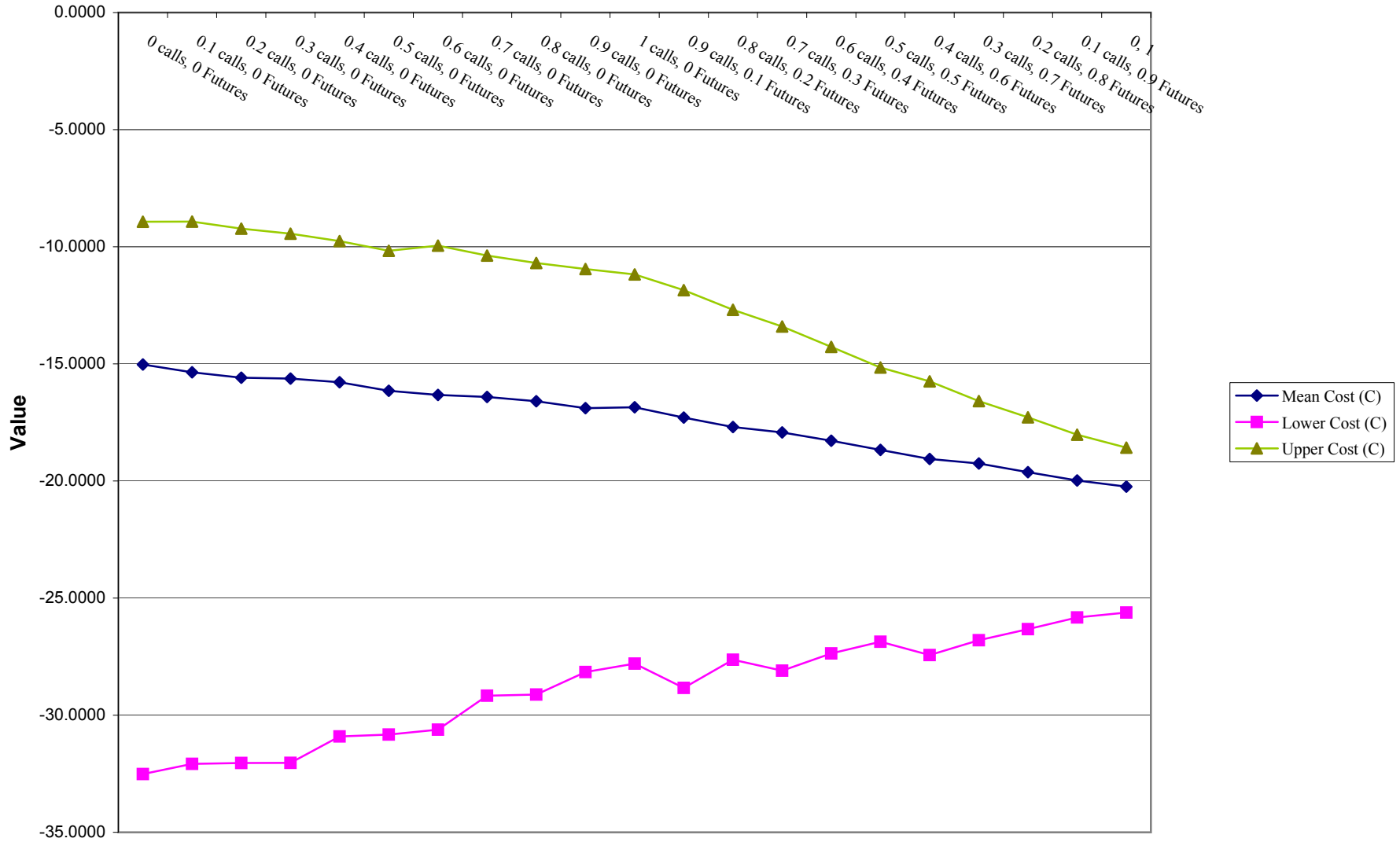
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<sup>2</sup>An alternative intuitive objective function, which gives rise to the same optimal solution, is *Downside-Risk Minimization*, based on the semi-variance criterion  $E(\tilde{E}) - \alpha \sqrt{E(\max\{[E(\tilde{E}) - \tilde{E}], 0\})^2}$ .

### Call and Future Cost



### Call and Future Cost



## Consumer Three-Way Payoff

## Consumer three-way payoff

