

Quantitative Methods for ALM

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Agenda

Quantitative Methods – Relevance to ALM

Quantitative Methods – Theory

- Valuation Principles

- Differential Calculus

- Statistical Measures

- Probability Theory

Quantitative Methods – Application

- Risk

- Duration

- Value at Risk

Quantitative Methods – Relevance to ALM

Risks relevant to ALM

- Liquidity risk
- Interest Rate risk
- Currency risk

Liquidity risk

- Stock Approach - employing ratios
 - Loan to asset
 - Loan to core deposit
 - Large liability to earning assets
 - Liquidity
- Flow Approach – Time bucket analysis

Interest rate risk

- Duration
 - Measures interest rate sensitivity
- Simulation
 - Modelling the bank's Interest rate sensitivity
- VaR
 - Choice of confidence level reflects risk appetite and cost of a loss exceeding the VaR
 - Can be used with stress testing to take care of event risk

Currency risk

- VaR approach to risk associated with exposure
- Use of derivative instruments as ALM tools

Relevance of QM to ALM

- Cash flows have to be seen in the context of time value of money, where compounding and discounting principles are applied
- Concepts like duration are better understood with mathematical ideas like differentiation
- VaR is derived from probability theory, distribution functions particularly, normal distribution

Quantitative Methods – Theory

Valuation Principles

- Time Value of Money
- Discounting and Present Value
- Cash Flows and Discounting Rate
- Continuous Discounting
- Merton's Valuation Models (based on Option Pricing)

Merton's Valuation Models

- Option Theoretic Approach to Debt
 - From the company's point of view, the debt obligation can be described by taking a long position in a put option; from the debt holder's (bank's) point of view, it can be described by writing a put option to the company
- Option Theoretic Approach to Equity
 - From the company's point of view, equity can be described by selling a call option to the equity holders; the position of equity holder is a long call on the firm's asset values

Differential Calculus

- Rate of change of a variable with respect to another variable - Denoted as dy/dx
- Positive indicates increasing function and negatives indicates decreasing function
- Second order differentiation gives rate of change of first order differential with respect to the variable – Denoted as d^2y/dx^2
- Positive indicates an increasing rate of change and negative indicates decreasing rate of change

Statistical Measures

- Measures of Central Tendency - Mean (AM, GM, HM), Mode & Median
- Measures of Dispersion – Range, Mean Deviation, Variance & Standard Deviation
- Measures of Skewness and Kurtosis
- Measures of Correlation and Regression

Probability Theory

- Probability – Definition & Measure
- Random Variable – Discrete & Continuous
- Probability Density Function
- Mathematical Expectation
- Distributions – Binomial & Normal

Bernoulli Distribution

Random variable X takes values 1 or 0
(Eg. Default)

$$P(X=1) = p$$

$$P(X=0) = 1-p$$

$$\text{Expectation} = p$$

$$\text{Variance} = p*(1-p)$$

Binomial Distribution

- Independent Bernoulli Trials
- No. of occurrences of one event (Default)
- Probability of r occurrences out of n trials

$${}^n C_r p^r (1-p)^{(n-r)}$$

- Expectation is $n \cdot p$
- Variance is $n \cdot p \cdot (1-p)$

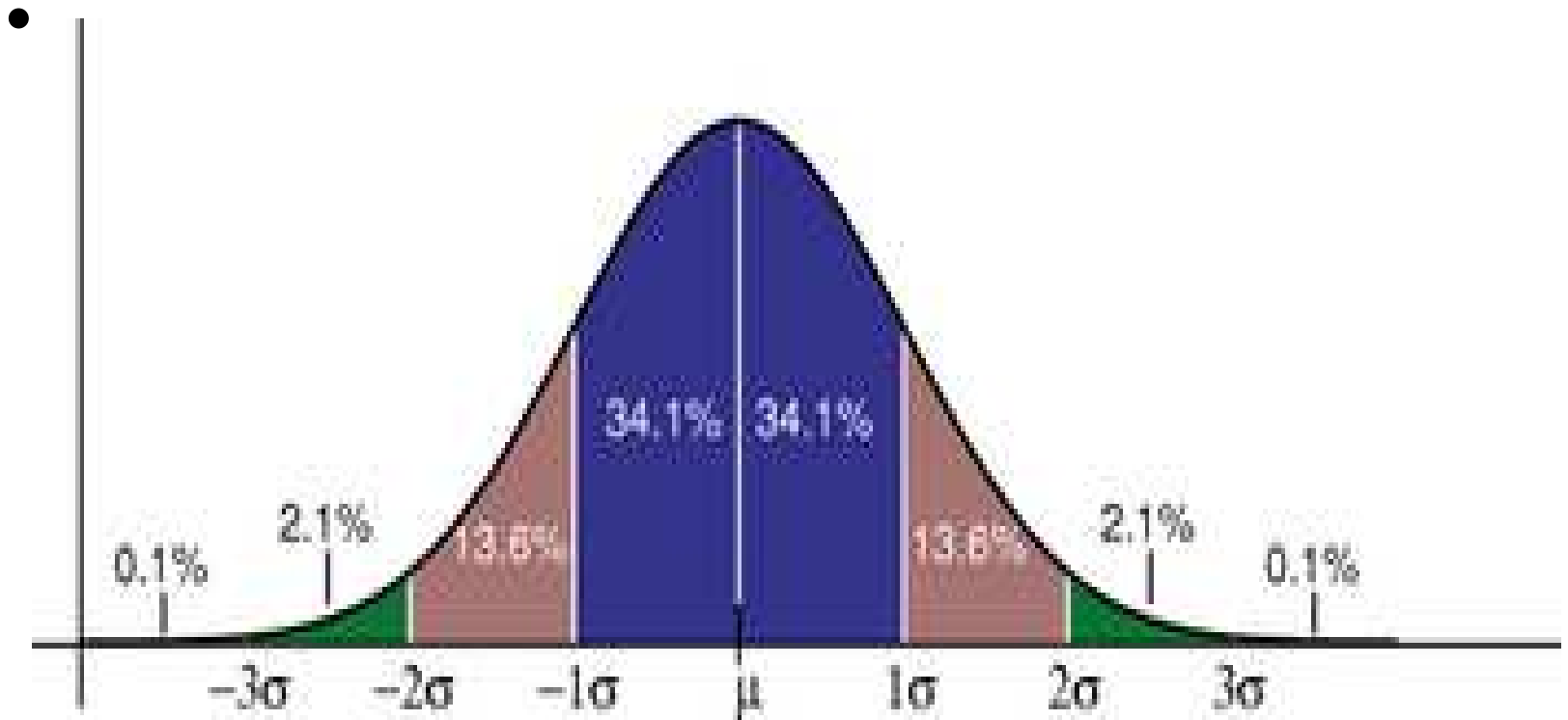
Normal Distribution

- Symmetric about mean
- Mean, Mode and Median are equal
- Skewness is zero
- Kurtosis is 3 (or 0)
- 66% of area lies between 1-sigma limits
- 95% of area lies between 2-sigma limits
- 99.7% of area lies between 3-sigma limits

Standard Normal

- If X follows Normal distribution with mean μ and standard deviation σ , then the variable $Z = (X - \mu) / \sigma$ follows Normal distribution with mean 0 and standard deviation 1
- Z is called standard normal variable
- Probabilities for various values of Z are readily available as tables in books

Standard Normal Curve



Excel Functions - Financial

- PV and NPV
- IRR and Rate
- PRICE
- YIELD
- ODDFPRICE, ODDLPRICE
- ODDFYIELD, ODDLYIELD
- DURATION, MDURATION

Excel Functions – Statistical

- AVERAGE, VAR, STDEV, SKEW, KURT
- CORREL, COVAR
- SLOPE, INTERCEPT
- NORMDIST, NORMINV
- NORMSDIST, NORMSINV
- LOGNORMDIST

Quantitative Methods - Applications

Measures of Risk

- Expectation or Arithmetic Mean can be a measure of Return
- Standard Deviation can be a measure of Risk / Volatility
- There are measures like Duration and Beta , which also are used as measures of Risk for a bond or an equity / portfolio

Bond Value Depends on . . .

- Yield (A decrease in a bond's yield will raise the bond's price by an amount that is greater in size than the corresponding fall in the bond's price that would occur if there were an equal-sized increase in the bond's yield)
- Coupon (The percentage change in a bond's price owing to a change in its yield will be smaller if its coupon rate is higher)
- Time to maturity (If a bond's YTM does not change over its life, then the size of its discount or premium will decrease at an increasing rate as its life gets shorter)

Measure of Risk of a Bond

- Since value of a bond depends on interest rate, it is important to measure the sensitivity of bond value to changes in interest rates.
- A measure of sensitivity or riskiness of bond is given by duration

Duration – What it is?

- **Average** period at which the amounts are due from a bond
- **Rate of change** of price of a bond with respect to YTM

Duration – How to Calculate?

$$D = \frac{\sum PV(C_t) * t + PV(FV) * n}{\sum PV(C_t) + PV(FV)}$$

1. Denominator is the price of the bond
2. Numerator is weighted average of time periods with PVs of cash flows as weights

Duration – How to Interpret it?

- Duration is a measure of risk. Higher the duration, riskier the bond
- Duration gives a relationship between percentage change in price and percentage change in YTM -

$$\frac{\Delta p}{p} = -D * \frac{\Delta(1 + y)}{(1 + y)} \text{ (approximately)}$$

[% change in price = - D * % change in (1 + YTM)]

Modified Duration

$$\text{MD} = \frac{D}{1 + y / p}$$

where D is duration

y is YTM

p is number of payments per year

[% change in price = - MD (Δ YTM / 100)]

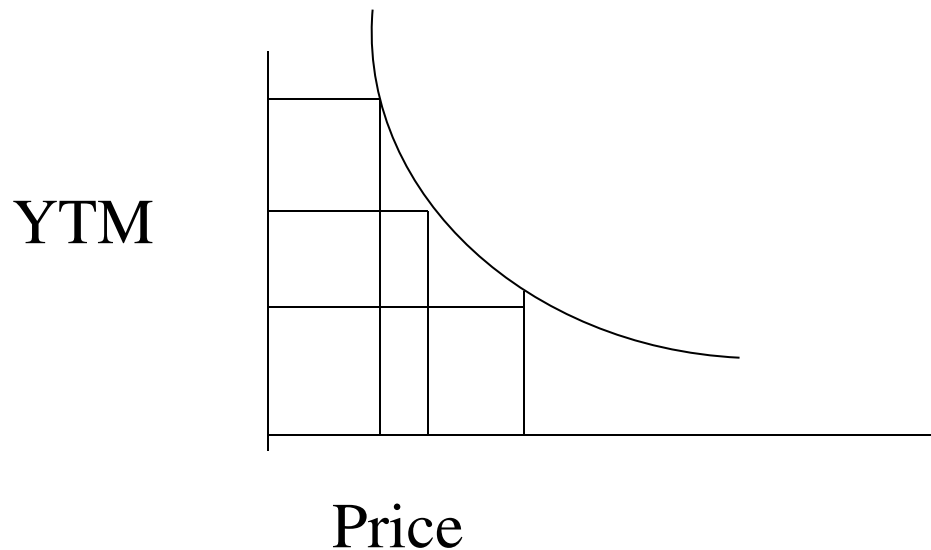
Duration of a Portfolio

- Let D_1 and D_2 be durations of two bonds
- Let w_1 and w_2 be percentage investments made in the two bonds, respectively
- Duration of portfolio is given as

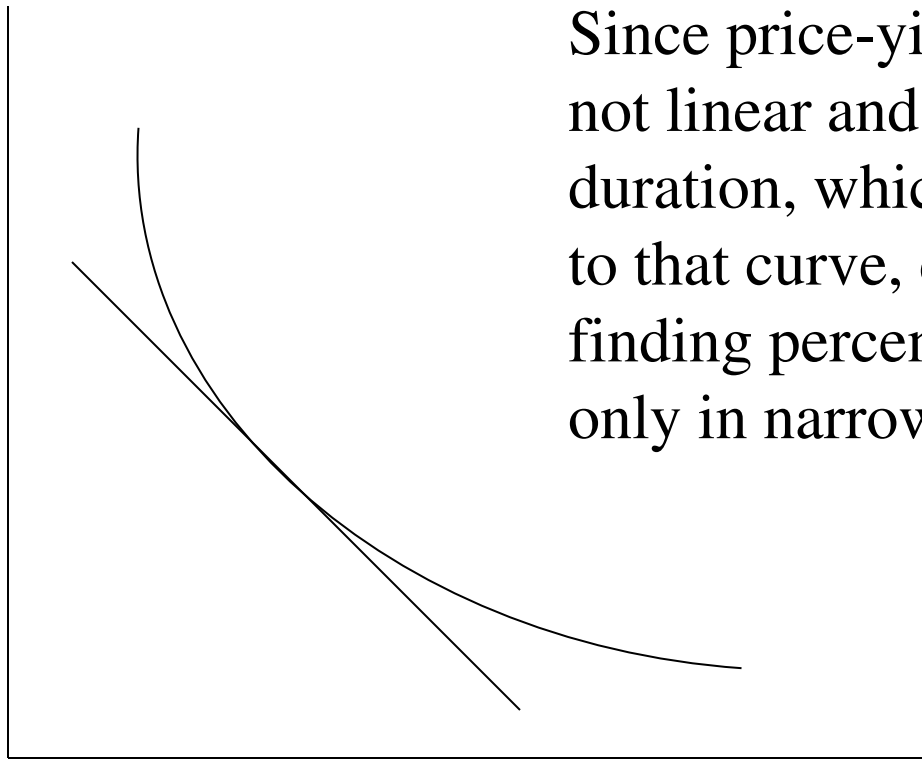
$$w_1D_1 + w_2D_2$$

Convexity

- Relationship between yield and price is convex



Duration & Convexity



Since price-yield curve is not linear and convex, duration, which is a tangent to that curve, can help in finding percentage changes only in narrow bands

VaR – What it is?

- 1) A forecast of a given **percentile**, usually in the lower tail, of the distribution of returns on a portfolio over some period; similar in principle to an estimate of the expected return on a portfolio, which is a forecast of the 50th percentile.

OR

- (2) An **estimate** of the level of loss on a portfolio which is expected to be equaled or exceeded with a given, small probability.

Var – How to calculate?

- $VaR = V * (c * \sigma + \mu)$

Where

V is the exposure

c = (1 – α) quantile of normal distribution

σ is standard deviation returns

μ is the expectation returns

- It is assumed that returns are normally distributed

VaR – Alternative Approaches

- Historical Simulation
- Monte-Carlo Simulation

VaR – How to Interpret it?

- It is assumed that returns are normally distributed (symmetric)
- Under the symmetric assumption, standard deviation can be used as a measure of risk
- By VaR analysis, we can say that there is 1 in 100, say, chance of losing X dollars over the holding period

Portfolio Diversification

- Immunity of a Bonds Portfolio
- Return and Risk of a Portfolio
- Optimization of a Portfolio
- Beta of a Portfolio
- Performance Measures

THANK YOU

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